Int. J. Heat Mass Transfer. Vol. 31, No. 6, pp. 1167–1172, 1988
Printed in Great Britain

0017–9310/88 $3.00+0.00
Pergamon Press plc

General theory of a convective nucleus of a fluid in unsteady state and in non-linear conditions

M. DE PAZ and G. SONNINO

Dipartimento di Fisica, Università di Genova, Via Dodecaneso, 33 16146 Genova, Italy

(*Received* 27 *April* 1987)

Abstract—A model of free convection in a fluid cylinder submitted to generic boundary conditions is developed. The fluid is subdivided into a boundary layer region and a nucleus moving with opposite velocities and the equations of Fourier, continuity and Navier–Stokes in the nucleus are solved exactly in terms of Fourier sums. The nucleus is linked to the boundary layer to provide the unknown function which solves completely this problem of convection. As an example the general solution is applied to the case of a temperature step which is found to travel undeformed through the nucleus (asymptotic solution).

1. INTRODUCTION

THE GENERAL problem of free convection in a fluid subjected to the gravity field and to a thermal gradient has been approached for a long time by studying almost exclusively steady-state situations [1].

The classical theory of an infinite hot vertical wall in contact with the fluid led to the formulation of the boundary layer region where the largest temperature and velocity gradients are observed. In this region the fluid behaviour is described by a quite complicated theory developed first by Prandtl [2].

A convenient way to study problems of convective linear instability is offered by thin fluid layers submitted to vertical temperature gradients giving rise to the classical Bénard cells, structures which can be obtained by maintaining the liquid for a long period of time in a steady-state condition.

A complete theoretical approach to non-linear instabilities and unsteady-state situations is still lacking.

In this paper we present a theory of the unsteady non-linear convective nucleus which holds under very general boundary conditions when a particular model of convection is assumed to work. The basic standpoint is that whenever convection occurs in a fluid there must be a boundary layer coupled with a complementary region called a nucleus, where the fluid flows with opposite velocity to counterbalance the mass flow in the boundary layer and give continuity to the system. In the gravity field the nucleus moves vertically collapsing into the boundary layer at the edges, regardless of the direction of the thermal gradient.

The validity of such a description is supported by experimental data and calculations presented in a previous paper [3].

The results of the theory are given in terms of general equations which can be applied to real systems, whenever the boundary condition functions are correctly recognized.

2. MODEL OF CONVECTION

The system under study is a cylinder of fluid, with its axis vertical, exchanging heat through its lateral wall.

We assume that convection is already established in the cylinder: this condition, from observations of the convection onset in transparent cylinders traced with coloured solutions, is reached in less than 1 s.

The volume of fluid is subdivided into three regions, as shown in Fig. 1.

(a) Boundary layer, a layer of liquid (average thickness δ) which moves vertically along the wall and reverses its velocity at the edges. Its temperature is very close to T_S, temperature of the lateral wall, through which heat is transferred by conduction from a thermostatic bath. The boundary layer acts as a 'convective heat source'.

(b) Central nucleus, defined as a cylinder of liquid moving inside the boundary layer with opposite velocity, independent of the radial distance from the axis and also independent of the level z. The temperature in the nucleus is constant at any point of a plane at level z, but is a function of z and time t.

(c) Intermediate region which joins smoothly regions (a) and (b). Along the vertical boundary this region is a very thin layer where the velocity and temperature variations in the radial direction are very large but not infinite. No fluid exchange occurs through this wall. At the top and at the bottom of the cylinder this region allows a smooth bending of the velocity vector from the boundary layer to the nucleus and vice versa. The overall volume of the intermediate region is negligible when compared to regions (a) and (b).

3. MATHEMATICAL TREATMENT OF THE MODEL

The fundamental equations of hydrodynamics applied to the described model reduce to (Appendix)

NOMENCLATURE

a	convection constant, $[2RA\mu\pi Gr^{1/3}Pr^{-2/3}]/[h(R-\delta)^2(G^*(h/2))^{1/3}]$
A	pure constant in a, independent of the nature of fluid
c_v	constant volume heat capacity
C_n	constants of the complete basis set $\phi_n(z)$, $in\pi/h$
e_{ij}	components of the strain tensor
f_1	initial temperature function defined in the interval $[0, \delta]$
f_2	initial temperature function defined in the interval $[h-\delta, h]$
g	constant of gravity
$G(z)$	initial temperature function defined in the interval $[\delta, h-\delta] = T(z,0)-T_S$
$G^*(z)$	initial temperature function defined in the interval $[-h, h]$
Gr	Grashof number, $\beta g G^*(h/2)h^3/v^2$
h	level of the liquid contained in the cylinder
K	thermal conductivity
K_1	pure constant in u_0
$\bar{k}$	unit vector along the z-axis
p	hydrostatic pressure
$\bar{\bar{P}}$	stress tensor
Pr	Prandtl number, v/χ
R	internal radius of the cylinder
t	time
t_0	initial time
T	temperature of the liquid in the nucleus
T_0	initial temperature (in the asymptotic solution)
T_S	surface temperature
$\bar{u}$	velocity
u_0	scalar velocity in the boundary layer (apart from the multiplicative function λ), $K_1 w^{-1} v\, Gr^{1/2}(1+0.49Pr^{2/3})^{-1/2}$
v	scalar velocity in the nucleus
v_0	scalar velocity at $t = t_0$
w	level in the liquid measured from the bottom, $h-z$
y^*	radial coordinate measured from the lateral surface of the cylinder towards the axis
z	level in the liquid measured from the top
z^*	normalized level in the liquid, $\pi z/h$.

Greek symbols

β	thermal expansion coefficient, $(-1/\rho(T_S))(\partial\rho/\partial T)$
δ	average thickness of the boundary layer
δ_{ij}	Kronecker delta function
$\bar{\nabla}$	gradient operator
$\bar{\nabla}\cdot$	divergency operator
μ	coefficient of viscosity
v	kinematic viscosity, μ/ρ
ρ	density of the liquid
ϕ	energy dissipated by viscous forces
χ	thermal diffusivity coefficient.

$$\frac{\partial T}{\partial t} - v\frac{\partial T}{\partial z} = \chi\frac{\partial^2 T}{\partial z^2} \quad \text{Fourier} \tag{1}$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial z}v \quad \text{Continuity} \tag{2}$$

$$-\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g \quad \text{Navier–Stokes.} \tag{3}$$

The boundary conditions are

$$T(z, t_0) = T_S + G(z) \tag{4}$$

$$v(t_0) = v_0 \tag{5}$$

$$\rho(z, t_0) = \rho(T_S)[1+\beta G(z)]. \tag{6}$$

Equation (4) is a general statement about the boundary temperature function $G(z)$ at the instant t_0.

Equation (5) expresses the condition that at $t = t_0$ the nucleus has already formed.

Equation (6) is the straightforward linear form of the density at temperature T.

In a purely convective system the term $\chi(\partial^2 T/\partial z^2) \simeq 0$ and equation (1) becomes indistinguishable from equation (2), since ρ is a function of T like equation (6).

This condition reduces the independent equations in the nucleus from three to two while there are three unknowns. The system becomes undetermined, but the third condition will be recovered when the nucleus and the boundary layer are interfaced through an appropriate continuity equation used as a subsidiary.

We look for solutions where variables z and t are separated into single terms

$$T = T_S + \sum_n f_n(t)\phi_n(z). \tag{7}$$

This approach is correct only if functions $f_n(t)$ and $\phi_n(z)$ *form a complete basic set.*

We can write

$$\frac{\partial T}{\partial t} = \sum_n \dot{f}_n(t)\phi_n(z)$$

$$v\frac{\partial T}{\partial z} = \sum_n f_n(t)v(t)\phi_n'(z)$$

and equation (1) becomes

$$\sum_n \dot{f}_n(t)\phi_n(z) - \sum_n f_n(t)v(t)\phi_n'(z) = 0. \tag{8}$$

Equation (8) is satisfied if the following hold for any n:

$$\dot{f}_n(t) - v(t) f_n(t) \frac{\phi_n'(z)}{\phi_n(z)} = 0 \tag{9}$$

i.e.

$$\frac{\phi_n'(z)}{\phi_n(z)} = C_n \quad \text{which gives} \quad \phi_n(z) = A_n \exp(C_n z). \tag{10}$$

Equation (9) becomes

$$v(t) = \frac{\dot{f}_n(t)}{C_n f_n(t)} \quad \text{(for any } n\text{)}. \tag{11}$$

Since $v(t)$ is independent of n, from equation (11) we must satisfy for any n and a fixed k the following condition:

$$\frac{\dot{f}_n(t)}{C_n f_n(t)} = \frac{\dot{f}_k(t)}{C_k f_k(t)} \tag{12}$$

which is equivalent to

$$\frac{\mathrm{d}f_n}{f_n} = \frac{C_n}{C_k} \frac{\mathrm{d}f_k}{f_k}. \tag{13}$$

Equation (13) upon integration gives

$$f_n(t) = [f_k(t)]^{C_n/C_k}. \tag{14}$$

From equations (11) and (14) with $k = 1$ (chosen at will)

$$v(t) = \dot{f}_n(t)/[C_n f_n(t)] = \dot{f}_1(t)/[C_1 f_1(t)] \tag{15}$$

$$f_n(t) = [f_1(t)]^{C_n/C_1}. \tag{16}$$

Equations (15) and (16) mean that in order to solve for $\phi_n(z) = A_n \exp(C_n z)$ we have to determine the set of constants C_n and A_n and *only one function* $f_1(t)$. The constants will be determined by means of the boundary conditions, while $f_1(t)$ will be obtained through the continuity condition applied at the interface between the nucleus and the boundary layer (see later).

We determine first the constants by observing that in equation (7) T must converge to T_S for $t \to \infty$ because the liquid must reach the temperature of the bath in contact with the lateral wall of the cylinder. This final condition is obtained by defining a properly shaped function $G^*(z)$ such as (at $t = t_0$)

$$G^*(z) = \begin{cases} 0 & \text{for} \quad -h \leqslant z < 0 \\ f_c^{(1)}(z) & \text{for} \quad 0 \leqslant z < \delta \\ G(z) & \text{for} \quad \delta \leqslant z \leqslant h-\delta \\ f_c^{(2)}(z) & \text{for} \quad h-\delta < z \leqslant h \end{cases} \tag{17}$$

where $f_c^{(1)}(z)$ and $f_c^{(2)}(z)$ are functions expressing the (small) temperature variations in the boundary layer region [4] and $G(z)$ is defined by equation (4).

At $t = t_0$ equation (7), using equation (10), becomes

$$T(z, t_0) = T_S + \sum_n f_n(t_0) A_n \exp(C_n z) = G^*(z) + T_S \tag{18}$$

which satisfies equations (1) and (2) with boundary condition (4) in the nucleus; moreover, at $t = 0$ it satisfies $f_c^{(1)}(z)$ and $f_c^{(2)}(z)$ in the boundary layer.

Since any function defined on a finite interval can be written as a Fourier series we conveniently put

$$\left.\begin{aligned} & G^*(z) = \sum_{-\infty}^{\infty} \alpha_n \exp(\mathrm{i}n\pi z/h) \\ & \text{with} \\ & \alpha_n = \frac{1}{2h} \int_{-h}^{h} G^*(z) \exp(-\mathrm{i}n\pi z/h)\,\mathrm{d}z. \end{aligned}\right\} \tag{19}$$

By comparing equation (18) with equations (19) we obtain

$$\begin{cases} \alpha_n = f_n(t_0) A_n \\ C_n = \mathrm{i}n\pi/h. \end{cases} \tag{20}$$

The result of equations (20) is astonishingly simple despite the complexity of the problem. In fact we are now able to calculate any f_n once f_1 is known.

From equations (16) and (20)

$$f_n(t) = f_1^n(t). \tag{21}$$

The resulting $T(z, t)$ in equation (7) is

$$T(z, t) = T_S + \sum_{n=-\infty}^{\infty} \alpha_n \frac{f_1^n(t)}{f_1^n(t_0)} \exp(\mathrm{i}n\pi z/h) \tag{22}$$

where $f_1(t)$ is the only unknown function. However, the velocity $v(t)$ must be a real function: as a consequence, from equations (15) and (20) we obtain

$$v(t) = \frac{\dot{f}_1(t)}{f_1(t)} \frac{h}{\mathrm{i}\pi}. \tag{23}$$

Since any complex function may be written as

$$f_1(t) = B(t) \exp[\mathrm{i}\psi(t)] \tag{24}$$

equation (23) becomes

$$v(t) = \frac{h}{\mathrm{i}\pi} \frac{\dot{B}(t) \exp[\mathrm{i}\psi(t)] + \mathrm{i}B(t)\dot{\psi}(t) \exp[\mathrm{i}\psi(t)]}{B(t) \exp[\mathrm{i}\psi(t)]}. \tag{25}$$

The condition of reality of equation (25) implies

$$\dot{B}(t) = 0 \quad \text{or} \quad B(t) = \text{constant} = B(t_0). \tag{26}$$

Equation (22) becomes

$$T(z, t) = T_S + \sum_{n=-\infty}^{\infty} \alpha_n \exp\{\mathrm{i}n[\psi(t) - \psi(t_0)]\} \exp(\mathrm{i}n\pi z/h). \tag{27}$$

Equation (23) becomes

$$v(t) = \frac{h}{\pi} \dot{\psi}(t). \tag{28}$$

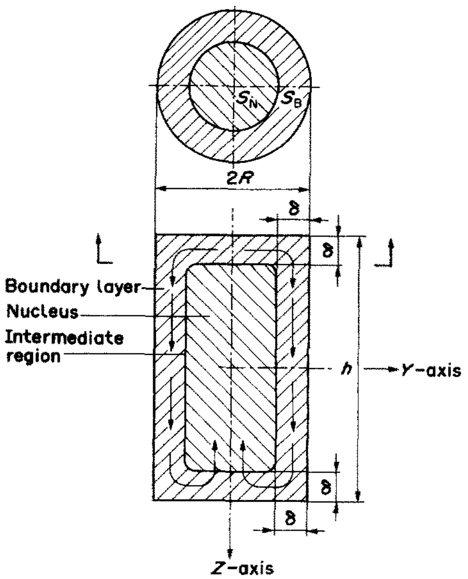

FIG. 1. Model of free convection in a vertical cylinder ($T_S < T$).

By putting $x(t) = \psi(t) - \psi(t_0)$ equations (27) and (28) become

$$T(z,t) = T_S + \sum_{n=-\infty}^{\infty} \alpha_n \exp[\mathrm{i}nx(t)] \exp(\mathrm{i}n\pi z/h)$$

$$v(t) = \dot{x}(t)h/\pi \tag{29}$$

where

$$x(t_0) = 0.$$

The constants α_n are given by equations (19)

$$\alpha_n = \frac{1}{2h}\int_{-h}^{h} G^*(z)\exp(-\mathrm{i}n\pi z/h)\,\mathrm{d}z.$$

If we put $z^* = z\pi/h$ we obtain

$$\alpha_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} G^*(z^*)\exp(-\mathrm{i}nz^*)\,\mathrm{d}z^*. \tag{30}$$

We have now to determine $x(t)$ with the condition $x(t_0) = 0$. This can be accomplished by the additional continuity equation at the boundary between the central nucleus and the boundary layer

$$\frac{\partial\rho}{\partial t} + \bar{\nabla}\cdot(\rho\bar{u}) = 0$$

$$\int \rho\bar{u}\,\mathrm{d}\bar{S} = -\int \frac{\partial\rho}{\partial t}\mathrm{d}V. \tag{31}$$

We calculate equations (31) by considering the volume V, as illustrated in Fig. 1, formed by the upper (or lower) cylindrical part of the boundary layer. This choice is compulsory to obtain information from the continuity equation not coincident with already known equations. We write

$$\int_{S_N} \rho\bar{u}_N\,\mathrm{d}\bar{S} + \int_{S_B} \rho\bar{u}_B\,\mathrm{d}\bar{S} = -\int_V \frac{\partial\rho}{\partial t}\mathrm{d}V$$

where the surfaces of integration S_N and S_B are chosen as described in Fig. 1 and u_N and u_B are the velocity vectors at S_N and S_B, respectively. Since the density ρ in V is almost equal to $\rho(T_S)$ we have

$$\frac{\partial\rho(T_S)}{\partial t} \simeq 0 \quad \text{and} \quad \int_V \frac{\partial\rho}{\partial t}\mathrm{d}V \simeq 0.$$

In scalar form and recalling that $u_N = v$ we obtain

$$\left.\begin{aligned} \int_{S_N} -\rho u_N\,\mathrm{d}S + \int_{S_B} \rho u_B\,\mathrm{d}S = 0 \\ -v\rho(\delta,t)(R-\delta)^2\pi + \rho(T_S)\int_{S_B} u_B\,\mathrm{d}S = 0. \end{aligned}\right\} \tag{32}$$

The integral in equations (31) may be calculated taking from the literature [5] the value of u_B as

$$u_B = u_0\lambda$$

where λ is a function of y/δ and

$$u_0 = K_1 w^{-1} v\,Gr^{1/2}(1+0.49Pr^{2/3})^{-1/2}$$

where K_1 is a proportionality constant, $w = h - z$, v is the kinematic viscosity, Gr the Grashof number and Pr the Prandtl number. This formulation of u_B is deduced for a laminar flow and by approximating the problem to a plane surface at constant T_S. However, experiments indicate that the correct combination of Gr and Pr in free turbulent convection is $Gr^{1/3}Pr^{-2/3}$ [5], i.e.

$$\int_{S_B} u_B\,\mathrm{d}S = 2\pi RA\mu\,Gr^{1/3}\,Pr^{-2/3}\left(\frac{T(\delta,t)-T_S}{G^*(h/2)}\right)^{1/3}$$

with

$$Gr^{1/3} = \left(\frac{\beta g h^3 G^*(h/2)}{v^2}\right)^{1/3}.$$

The velocity v is given by

$$v = \frac{2RA\mu\rho(T_S)\,Gr^{1/3}\,Pr^{-2/3}}{(R-\delta)^2\rho(\delta,t)[G^*(h/2)]^{1/3}}[T(\delta,t)-T_S]^{1/3}. \tag{33}$$

We observe that in equation (33) the ratio $\rho(T_S)/\rho(\delta,t) \simeq 1$ to a very good approximation and, recalling that $v = \dot{x}h/\pi$, we have

$$\dot{x} = a[T(\delta,t)-T_S]^{1/3} \tag{34}$$

where

$$a = \frac{2RA\mu\pi\,Gr^{1/3}\,Pr^{-2/3}}{h(R-\delta)^2[G^*(h/2)]^{1/3}} \tag{35}$$

and

$$x(t_0) = 0. \tag{36}$$

From equations (29) we calculate

$$T(\delta, t) = T_S + \sum_{n=-\infty}^{\infty} \alpha_n \exp\{in[x(t)+\pi\delta/h]\}$$

or

$$T(\delta, t) - T_S = G^*[x(t)h/\pi + \delta].$$

By substituting in equation (34) we obtain

$$\dot{x} = aG^{*1/3}[x(t)h/\pi + \delta]$$
$$x(t_0) = 0. \tag{37}$$

Equation (34) has the general solution (integral form)

$$\int_0^{x(t)} \frac{dy}{G^{*1/3}(yh/\pi + \delta)} = a(t-t_0). \tag{38}$$

The entire set of equations is now solved and can be applied to various practical and theoretical situations by changing the boundary conditions and varying the choice of G^* at will. In the following section we report a solution which is asymptotic and in our opinion is important to clarify the physical meaning of a steep temperature gradient applied to a cylinder of fluid obeying our model.

Once the solution of equation (38) $x(t)$ is known, we observe that the hydrostatic pressure in the cylinder is given by the following formula (Appendix):

$$p = (h\ddot{x}/\pi + g)\int_0^z \rho(T_S)\{1-\beta[T(z,t)-T_S]\}\,dz + P_0 \tag{39}$$

where P_0 is the hydrostatic pressure at $z = 0$.

4. ASYMPTOTIC SOLUTION

We choose as the boundary function $G^*(z) = (R_0 - T_S)\theta(z)$ where $\theta(z)$ is a double step function: $\theta(z) = 0$ for $-h \leqslant z < \delta$, $h-\delta < z \leqslant h$; $\theta(z) = 1$ for $\delta \leqslant z \leqslant h-\delta$. At $t = 0$ the nucleus is at $T = T_0$ and the boundary layer is at $T = T_S$.

The differential equations (37) become

$$\begin{cases} \dot{x} = a(T_0 - T_S)^{1/3}\theta[x(t)h/\pi + \delta] \\ x(t_0) = 0 \end{cases} \tag{40}$$

where

$$a = \frac{2RA\mu\pi\, Gr^{1/3}\, Pr^{-2/3}}{h(R-\delta)^2(T_0 - T_S)^{1/3}}.$$

The solution of equation (40) is

$$x(t) = \begin{cases} a(T_0 - T_S)^{1/3}(t-t_0) & \text{for} \quad t_0 \leqslant t \leqslant t_0 + \dfrac{\pi(1-2\delta/h)}{a(T_0-T_S)^{1/3}} \\ (1-2\delta/h) & \text{for} \quad t > t_0 + \dfrac{\pi(1-2\delta/h)}{a(T_0-T_S)^{1/3}}. \end{cases} \tag{41}$$

As a consequence

$$v(t) = \frac{h}{\pi}\dot{x}(t)$$

or

$$v(t) = \begin{cases} \dfrac{ha(T_0-T_S)^{1/3}}{\pi} & \text{for} \quad t_0 \leqslant t \leqslant t_0 + \dfrac{\pi(1-2\delta/h)}{a(T_0-T_S)^{1/3}} \\ 0 & \text{for} \quad t > t_0 + \dfrac{\pi(1-2\delta/h)}{a(T_0-T_S)^{1/3}}. \end{cases}$$

The temperature function is determined by calculating the α_n coefficients by means of equation (30)

$$\alpha_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} (T_0 - T_S)\theta(z^*)\exp(-inz^*)\,dz^* \tag{42}$$

where

$\theta(z^*) = 1$ for $\delta\pi/h \leqslant z^* \leqslant (1-\delta/h)\pi$

and

$\theta(z^*) = 0$ for $-\pi \leqslant z^* < \pi\delta/h$ and $\pi(1-\delta/h) < z^* \leqslant \pi$.

From equation (41)

$$\alpha_n = (T_0 - T_S) \times \frac{i[(-1)^{n+1}\exp(-in\pi\delta/h) + \exp(in\pi\delta/h)]}{2n\pi}. \tag{43}$$

As a consequence the temperature function is

$$T(z,t) = T_S + \sum_{n=-\infty}^{\infty} \alpha_n \exp[in(x(t) + \pi z/h)] \tag{44}$$

where α_n are given by equation (43) and $x(t)$ by equations (41).

Equation (44) is a step function which travels through the nucleus and is obviously a limiting case of convection. The real case is obtained by replacing the $\theta(z)$ function with an appropriate smoothed step function which will be described in a forthcoming paper.

5. CONCLUSIONS

The equations presented here apply to a liquid contained in a cylindrical vessel, but, as we will show in another paper, a more general formulation for any form of the container is possible, provided that the appropriate geometrical and boundary layer functions are used. As far as the convective nucleus is concerned no cylinder thickness or height is required to provide a valid range to our equations. The only limit of this theory is in the approximations involved in the boundary layer equations, if the convective nucleus exists.

Acknowledgements—We are grateful to Prof. Andrea Levi for criticism and suggestions. We are indebted to Prof. Mario Bertero for his contribution in finding the asymptotic solution. This work was partly supported by Ministero della Pubblica Istruzione.

REFERENCES

1. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*. Clarendon Press, Oxford (1961).
2. L. Prandtl, Zur berechnung der grenzschichten, *Z. Angew. Math. Mech.* **18,** 77 (1938).
3. M. De Paz, M. Pilo and G. Sonnino, Non-linear, unsteady free convection in a vertical cylinder submitted to a horizontal thermal gradient: measurements in water between 6 and 21°C and a theoretical model of convection, *Int. J. Heat Mass Transfer* **30,** 289–295 (1986).
4. G. E. Gadd, C. W. Jones and E. J. Watson, Approximate methods of solution. In *Fluid Motion Memoirs, Laminar Boundary Layers* (Edited by L. Rosenhead). Clarendon Press, Oxford (1963).
5. H. Mouton et H. De Roeck, Convection naturelle au sein d'un liquide verticale fermée soumise à un changement brusque de température ambiante, *Int. J. Heat Mass Transfer* **20,** 627 (1977).

APPENDIX

The fundamental equations of hydrodynamics are:

$$\rho\frac{\partial}{\partial t}(c_v T)+\rho\bar{u}\cdot\bar{\nabla}(c_v T) = \bar{\nabla}\cdot(K\bar{\nabla}T)-p\bar{\nabla}\cdot\bar{u}+\phi \quad \text{(Fourier)}$$

$$\frac{\partial\rho}{\partial t}+\bar{\nabla}\cdot(\rho\bar{u}) = 0 \quad \text{(Continuity)}$$

$$\rho\frac{\partial\bar{u}}{\partial t}+\rho(\bar{u}\cdot\bar{\nabla})\bar{u} = \rho\bar{g}+\bar{\nabla}\cdot\bar{\bar{P}} \quad \text{(Navier–Stokes).}$$

By choosing the z-axis as in Fig. 1 and putting $T = T(z, t)$ and $\bar{u} = -v(t)\bar{k}$, these equations become equations (1)–(3).

The Navier–Stokes equation gives, for the components x and y (with the sum rule convention and by numbering x, y, z as x_1, x_2, x_3)

$$\frac{\partial P_{1J}}{\partial x_J} = 0; \quad \frac{\partial P_{2J}}{\partial x_J} = 0. \tag{A1}$$

As $P_{ij} = -p\delta_{ij}$, equations (A1) become $\partial p/\partial x = \partial p/\partial y = 0$, i.e. $p = p(z, t)$.

THEORIE GENERALE DU NOYAU CONVECTIF D'UN FLUIDE DANS DES CONDITIONS D'ETAT NON PERMANENT ET DE NON LINEARITE

Résumé—On développe un modèle de convection naturelle dans un cylindre de fluide soumis à des conditions limites génériques. Le fluide est divisé en une région de couche limite et en un noyau se déplaçant avec des vitesses opposées et les équations de Fourier, de continuité et de Navier–Stokes dans le noyau sont résolues exactement en fonction des sommes de Fourier. Le noyau est couplé à la couche limite pour fournir la fonction inconnue qui résout complètement ce problème de convection. Pour illustration, la solution générale est appliquée au cas d'un échelon de température lequel se déplace sans déformation à travers le noyau (solution asymptotique).

ALLGEMEINE THEORIE EINES FLUIDEN KONVEKTIONSKERNES UNTER INSTATIONÄREN NICHT-LINEAREN BEDINGUNGEN

Zusammenfassung—Es wird ein Modell der freien Konvektion in einem Fluidzylinder entwickelt. Das Fluid wird in ein Grenzschicht-Gebiet und ein Kern-Gebiet unterteilt, welche sich mit entgegengerichteter Geschwindigkeit bewegen. Die Fourier-, Kontinuitäts- und Navier–Stokes-Gleichungen für den Kernbereich werden in Form Fourier-scher Summen exakt gelöst. Der Kern wird mit der Grenzschicht gekoppelt, was die unbekannte Funktion und damit die vollständige Lösung dieses Konvektions-Problems ergibt. Als Beispiel wird die allgemeine Lösung auf den Fall eines Temperatursprunges angewandt, wobei sich ergibt, daß dieser unverändert durch den Kern hindurch läuft (asymptotische Lösung).

ОБЩАЯ ТЕОРИЯ ТЕЧЕНИЯ ЯДРА ЖИДКОСТИ ПРИ СВОБОДНОЙ КОНВЕКЦИИ ДЛЯ НЕСТАЦИОНАРНЫХ И НЕЛИНЕЙНЫХ УСЛОВИЙ

Аннотация—Разработана модель свободной конвекции в цилиндре, заполненном жидкостью, при произвольных граничных условиях. Объём, занятый жидкостью, подразделяется на область пограничного слоя и ядро, движущиеся с противоположными скоростями; уравнения Фурье, неразрывности и Навье–Стокса для ядра точно решаются с помощью суммирования гармоник ряда Фурье. Решение для ядра связывается с решением для пограничного слоя. В качестве примера дано общее решение задачи для случая скачка температуры, который, как оказалось, не изменяется для ядра (асимптотическое решение).